Engineering Notes

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Heave Motion of Air Cushion Vehicles

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Nomenclature

a	= wave amplitude
A_0	= equilibrium leakage area
A_p	= planform area
b	= beam
C_n	= orifice coefficient
h	= heave coordinate
H	= Heaviside step function
h_r	= height relative to surface
k	= wavenumber
K	= outward slope of skirt
ℓ	= craft length
m	= craft mass
P	= cushion pressure
P_0	= equilibrium cushion pressure
$Q_{\text{in,out}}$	= flow in, out of cushion
t	= time
U	= speed of craft
V	= cushion volume
x	= coordinate direction parallel to craft axis
η	= wave surface elevation
λ	= wave length
ρ	= air density
ω	= wave frequency
ω_e	= encounter frequency

Introduction

THE problem considered in this paper is the prediction of heave response for a fully skirted air cushion vehicle as the craft travels at constant speed and course in head seas made up of sinusoidal, deepwater waves. An analytical approach is taken in which the dynamic equations are derived from first principles. The theoretical predictions, presented as a closed-form solution, take into account nonlinearities associated with restoring force terms.

The seakeeping problem for ACV's has usually been approached using computer simulations[1-3] or experimental studies[4,5] with comparably little being done using analytical methods. This has been due to the great complexity of the equations involved so that numerical solutions or semi-empirical methods are resorted to, except for relatively simple cases in which linear models have been developed.[6-8] Despite the difficulties involved, closed-form solutions are advantageous for providing physical insight and as a direct means of evaluating the effect of design particulars on the vehicle dynamics.

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In this paper, the heave motion alone is considered, which reduces the complexity of the equation of motion and the conservation of air mass expression. These relations are used to solve for the heave coordinate in terms of the wave parameters and the craft speed and design specifications. The theory is then discussed in terms of the heave motion frequency response and is compared with data obtained by experiment using a scale model in a towing tank.

Mathematical Analysis

The ACV considered is traveling at constant speed U while encountering head seas consisting of regular, sinusoidal waves of surface height

$$\eta(x,t) = a \sin(kx + \omega_e t) \tag{1}$$

The wave amplitude is a; $k = 2\pi/\lambda$ is the wavenumber and $\omega_e = \omega + kU$ is the encounter frequency. The Froude-Kryloff hypothesis will be employed; that is, the waves are assumed noncompliant.

The equation of motion for the heave response governs the heave coordinate h (measured from equilibrium craft height obtained when $a=0$). The vertical rate of momentum change equals the force in the vertical direction which is taken as the cushion pressure acting over the cushion's external projected area:

$$m\ddot{h} = (P - P_0)A_p + P_0 K \int_{\substack{\text{craft}\\ \text{periphery}}} \mathrm{d}s(\eta - h)H(\eta - h) \tag{2}$$

The first forcing term accounts for pressure changes from the equilibrium value, while the second term on the right-hand side represents the hydrodynamic forces on the outwardly slanting skirt. The seal system is assumed to be completely flexible and without inertia so that the hydrodynamic pressure approximates the equilibrium cushion pressure. The effect of skirt contact is then to increase projected area, and the Heaviside step function is used to restrict contributions to those in which contact has actually been made. The cushion air pressure is determined from a conservation of mass relation for the air flow. Because of the large leakage of ACV's the effects of compressibility and, hence, absolute pressure may be neglected. Employing this assumption, the rate of cushion volume change must equal the net flow into the cushion:

$$\frac{\mathrm{d}V}{\mathrm{d}t} = Q_{\text{in}} - Q_{\text{out}} \tag{3}$$

The flow in is due to air supplied by fans. A linearized flow-pressure relation is used to represent the "fan map" near the equilibrium operating point and is given by

$$Q_{\text{in}} = Q_0 - Q_I(P - P_0) \tag{4}$$

where

$$Q_I = -\left(\frac{\partial Q_{\text{in}}}{\partial P}\right) > 0 \tag{5}$$

The flow out of the main cushion is due to leakage beneath the surrounding skirt. An orifice-coefficient approach will be used so that

$$Q_{\text{out}} = C_n \left[A_0 + \int \mathrm{d}s(h-\eta)H(h-\eta) \right] \sqrt{\frac{2P}{\rho}} \tag{6}$$

The leakage area includes a part present at equilibrium, A_0, and additional area due to gap openings between skirt and surface while encountering waves.

For small fluctuations about the equilibrium point, Eq. (6) may be linearized to yield the expression

$$Q_{\text{out}} = C_n A_0 \sqrt{\frac{2P_0}{\rho}} + C_n A_0 \frac{(P-P_0)}{\sqrt{2\rho P_0}} + C_n \left\{ \int \mathrm{d}s(h-\eta)H(h-\eta) \right\} \sqrt{\frac{2P_0}{\rho}} \tag{7}$$

Cushion volume changes are due to variations in cushion height and the volume occupied by the wave. The rate of cushion volume change is then

$$\frac{\mathrm{d}V}{\mathrm{d}t} = A_p \dot{h} - \frac{2ab\omega_e}{k} \sin\frac{k\ell}{2} \cos\omega_e t \tag{8}$$

Combining Eqs. (2), (3), (4), (7), and (8) to eliminate the pressure P, the equation of motion becomes

$$m\ddot{h} + \frac{A_p^2}{Q_1 + \frac{C_n A_0}{\sqrt{2\rho P_0}}} \dot{h} + \int_{\substack{\text{craft}\\ \text{periphery}}} \mathrm{d}s \left\{ \frac{C_n A_p \sqrt{\frac{2P_0}{\rho}}}{Q_1 + \frac{C_n A_0}{\sqrt{2\rho P_0}}} (h-\eta)H(h-\eta) + P_0 K(h-\eta)H(\eta-h) \right\} = \frac{2A_p ab\omega_e \sin\frac{k\ell}{2}}{k\left[Q_1 + \frac{C_n A_0}{\sqrt{2\rho P_0}}\right]} \cos\omega_e t \tag{9}$$

By examining the third term on the left-hand side of the above equation, it is seen that the stiffness is given by terms which are piecewise linear in the relative height $h_r = h - \eta$. They have the form

$$F(h_r) = k_1 h_r H(h_r) + k_2 h_r H(-h_r) = k_2 h_r + (k_1 - k_2) h_r H(h_r)$$

where

$$k_1 = \frac{C_n A_p \sqrt{\frac{2P_0}{\rho}}}{Q_1 + \frac{C_n A_0}{\sqrt{2\rho P_0}}} \quad \text{and} \quad k_2 = P_0 K \tag{10}$$

This results from the fact that when the relative height is positive, a restoring force is developed due to pressure drop accompanying cushion drainage. When the relative height is negative, on the other hand, the restoring force is caused by an increase in projected area. Thus different physical mechanisms operate depending on the relative height. In general, cushion drainage is more effective so that $k_1 > k_2$.

This nonlinearity will be dealt with by approximating the piecewise linear term with an expansion to second order in relative height:

$$h_r H(h_r) \simeq C_0 + C_1 h_r + C_2 h_r^2 \tag{11}$$

The C's cannot be found as they would be for an ordinary Taylor's series expansion since $h_r H(h_r)$ has a discontinuous derivative at $h_r = 0$. Instead, C_0, C_1 and C_2 are determined such that if h_r is approximated by its first harmonic, the Fourier series of the left- and right-hand sides of Eq. (11) will have the same first three terms. Equation (9) may then be rewritten as

$$\ddot{h} + B\dot{h} + \omega_0^2 h = c\cos(\omega_e t - \beta) - \Delta \int \mathrm{d}s [C_0 + C_2 h_r^2] \tag{12}$$

where

$$B = \frac{A_p^2}{m\left[Q_1 + \frac{C_n A_0}{\sqrt{2\rho P_0}}\right]}, \qquad \omega_0^2 = \frac{(k_1 + k_2)(\ell + b)}{m}$$

$$c = \left\{ \left(\frac{2A_p ab\omega_e \sin\frac{k\ell}{2}}{mk\left[Q_1 + \frac{C_n A_0}{\sqrt{2\rho P_0}}\right]} \right)^2 + \left[\frac{(k_1+k_2)a\left(b\cos\frac{k\ell}{2} + \frac{2}{k}\sin\frac{k\ell}{2}\right)}{m} \right]^2 \right\}^{1/2},$$

and

$$\Delta = \frac{k_1 - k_2}{m}$$

This allows a solution to be obtained, for encounter frequencies near ω_0, using perturbation techniques similar to those used in connection with the familiar hard/soft spring harmonic oscillator problem. The result is

$$h = C\cos(\omega_e t - \phi) + \frac{\Delta C(\ell + b)}{\pi \omega_e^2} \left[-2 - \frac{4}{9}\cos(\omega_e t - \phi) + \frac{4}{9}\cos 2(\omega_e t - \phi) \right] \tag{13}$$

where

$$C = \left[\frac{c^2}{(\omega_e^2 - \omega_0^2)^2 + B^2 \omega_e^2} \right]^{1/2}$$

is the amplitude of the oscillation about the mean elevation and

$$\phi = \tan^{-1} \left\{ \frac{9(k_1+k_2)\left[b\cos\frac{k\ell}{2} + \frac{2}{k}\sin\frac{k\ell}{2}\right]}{\frac{2A_p ab\omega_e \sin\frac{k\ell}{2}}{k\left[Q_1 + \frac{C_n A_0}{\sqrt{2\rho P_0}}\right]}} \right\} - \sin^{-1}\left(\frac{BC\omega_e}{c}\right)$$

is the phase lag of the response. The first term on the right hand side of the solution for h, $C\cos(\omega_e t - \phi)$, and its amplitude C coincide with the solution to a linearized version of the problem (not restricted to frequencies near ω_0) in which the term preceded by Δ in Eq. (12) is neglected.

Discussion

The solution consists of a "linear" part $C\cos(\omega_e t - \phi)$, and a part due to the nonlinearity in the restoring force. The nonlinear portion of the solution includes a downwards shift in the mean elevation of the trajectory as the craft responds to cushion drainage through skirt-surface gaps at wave troughs. First and second harmonic terms are also produced.

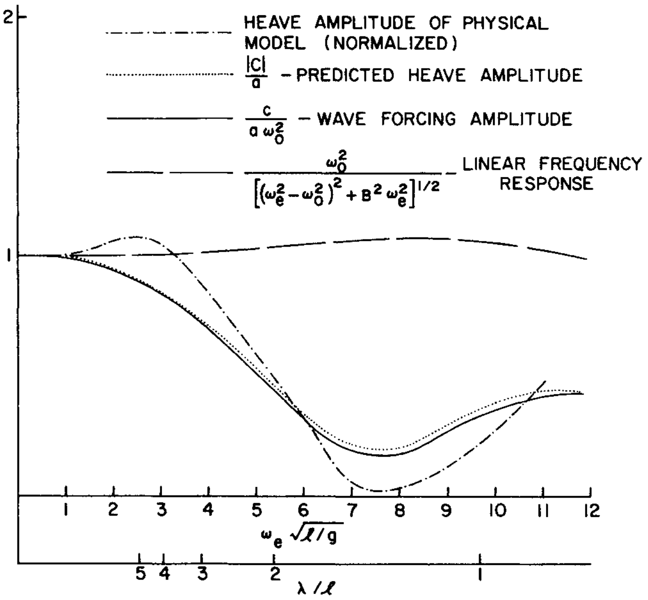

Fig. 1 Craft response at $U/\sqrt{\ell g} = 1.15$.

The analytical solution has been applied to 7/100 scale model of the JEFF(A) with the results shown in Fig. 1. The predicted value of the normalized amplitude of the solution ($= |C|/a$) is plotted vs encounter frequency and wavelength for a Froude number of 1.15. The magnitude of the wave forcing term and the linear frequency response are also shown separately since these are the factors which generate the heave response. It may be seen that the heave response of the vehicle is controlled by the form of the wave forcing curve, while the predicted linear frequency response is quite flat at this speed for wavelengths greater than the craft length.

For comparison, an experimentally determined heave response obtained from towing tank tests as presented in Ref. 3 is included. The experimental curve behaves roughly in the same manner as the theoretical prediction. This suggests that the model may be relied upon to explain physical mechanisms and the influence of design particulars, though not in precise quantitative terms.

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Technical Comments

Comments on "Controlling the Separation of Laminar Boundary Layers in Water: Heating and Suction"

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THESE comments on Ref. 1 by Aroesty and Berger are aimed at indicating that Aroesty and Berger: 1) solved incorrect describing equations; 2) made unacceptable assumptions regarding the variation of the fluid properties; and 3) performed a series of mathematical approximations which led to entirely erroneous results.

With the notation of Ref. 1, the variable fluid properties Falkner-Skan equations are

$$(Nf'')' + ff'' + \beta(R - f'^2) = 0 \quad (1)$$

$$(Ng'/Pr)' + fg' = 0 \quad (2)$$

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For heated water boundary layers, the density ratio R may be taken as unity. Aroesty and Berger write the energy equation as

$$g'' + Pr_\infty fg' = 0 \quad (3)$$

While this is very convenient for the approximations which Aroesty and Berger later make, it is unacceptable in problems dominated by fluid properties variations.

Further violation of the physics of the problem results from the assumption of the constant N. The extensive parametric study of Ref. 2 showed that variation of fluid properties plays a critical role in boundary-layer separation. In view of this conclusion, the assumption of $N = \text{const}$ renders the paper of Aroesty and Berger invalid.

After a questionable reduction of the differential equations and crude mathematical approximations, Aroesty and Berger conclude that the increment of surface temperature necessary to prevent separation may be expressed as

$$\Delta T_{\min} = -1000(\beta + 0.1988) \quad (4)$$

The computer program used in Ref. 2 was exercised some time ago in a parametric study of separating heated water boundary layers. This program solved the inverse Falkner-Skan problem ($f''(0) = 0$, β to be found) exactly for a complete variation of fluid properties and the results for $T_\infty = 60°\text{F}$ may be expressed as

$$\Delta T_{\min} = 2.55(1000 * |\beta + 0.1988|)^{1.075} \quad (5)$$